




Introduction of bar models for additive reasoning: Year 1/2

<p>Part-whole problems Present calculations in this way sometimes.</p>	<p><i>(Whole unknown)</i> Dev has 5 red marbles and 8 blue marbles. How many marbles does he have?</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">?</td></tr> <tr><td style="text-align: center;">5 red</td><td style="text-align: center;">8 blue</td></tr> </table>	?		5 red	8 blue	<p><i>(Part unknown)</i> Dev has 13 marbles. Five are red and the rest are blue. How many blue marbles does Dev have?</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">13</td></tr> <tr><td style="text-align: center;">5 red</td><td style="text-align: center;">? blue</td></tr> </table>	13		5 red	? blue													
?																							
5 red	8 blue																						
13																							
5 red	? blue																						
<p>Joining problems Present addition calculations in this way sometimes: $3 + 5 = ?$</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">?</td></tr> <tr><td style="text-align: center;">3</td><td style="text-align: center;">5</td></tr> </table> <p>$3 + ? = 8$</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">8</td></tr> <tr><td style="text-align: center;">3</td><td style="text-align: center;">?</td></tr> </table>	?		3	5	8		3	?	<p><i>(Result Unknown)</i> Dev had 5 marbles. Juan gave him 8 more marbles. How many marbles does Dev have now?</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">Dev ?</td></tr> <tr><td style="text-align: center;">Dev 5</td><td style="text-align: center;">Juan 8</td></tr> </table>	Dev ?		Dev 5	Juan 8	<p><i>(Change Unknown)</i> Dev has 5 marbles. How many more marbles does he need to have 13 marbles altogether?</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">13</td></tr> <tr><td style="text-align: center;">Dev 5</td><td style="text-align: center;">?</td></tr> </table>	13		Dev 5	?	<p><i>(Start Unknown)</i> Dev had some marbles. Juan gave him 5 more marbles. Now he has 13 marbles. How many marbles did Dev have to start with?</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">13</td></tr> <tr><td style="text-align: center;">Dev ?</td><td style="text-align: center;">Juan 5</td></tr> </table>	13		Dev ?	Juan 5
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3	?																						
Dev ?																							
Dev 5	Juan 8																						
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Dev 5	?																						
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Dev ?	Juan 5																						
<p>Separating problems Present subtraction calculations in this way sometimes: $8 - 5 = ?$</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">8</td></tr> <tr><td style="text-align: center;">?</td><td style="text-align: center;">5</td></tr> </table> <p>$8 + ? = 5$</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">8</td></tr> <tr><td style="text-align: center;">5</td><td style="text-align: center;">?</td></tr> </table>	8		?	5	8		5	?	<p><i>(Result Unknown)</i> Dev had 13 marbles. He gave 5 to Juan. How many marbles does Dev have left?</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">Dev 13</td></tr> <tr><td style="text-align: center;">Juan 5</td><td style="text-align: center;">?</td></tr> </table>	Dev 13		Juan 5	?	<p><i>(Change Unknown)</i> Dev had 13 marbles. He gave some to Juan. Now he has 5 marbles left. How many marbles did Dev give to Juan?</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">Dev 13</td></tr> <tr><td style="text-align: center;">Juan ?</td><td style="text-align: center;">Dev 5</td></tr> </table>	Dev 13		Juan ?	Dev 5	<p><i>(Start Unknown)</i> Dev had some marbles. He gave 5 to Juan. Now he has 8 marbles left. How many marbles did Dev have to start with?</p> <table border="1" style="margin: 10px auto;"> <tr><td colspan="2" style="text-align: center;">Dev ?</td></tr> <tr><td style="text-align: center;">Juan 5</td><td style="text-align: center;">Dev 8</td></tr> </table>	Dev ?		Juan 5	Dev 8
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5	?																						
Dev 13																							
Juan 5	?																						
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Juan ?	Dev 5																						
Dev ?																							
Juan 5	Dev 8																						

<p>Comparing problems</p>	<p><i>(Difference unknown)</i> Dev has 13 marbles. Juan has 5 marbles. How many more marbles does Dev have than Juan?</p> <table border="1" data-bbox="543 321 900 436"> <tr> <td colspan="2">Dev 13</td> </tr> <tr> <td>Juan 5</td> <td>?</td> </tr> </table>	Dev 13		Juan 5	?	<p><i>(Smallest part unknown)</i> Dev has 13 marbles. He has 5 more marbles than Juan. How many marbles does Juan have?</p> <table border="1" data-bbox="1029 321 1386 436"> <tr> <td colspan="2">Dev 13</td> </tr> <tr> <td>Juan ?</td> <td>5</td> </tr> </table>	Dev 13		Juan ?	5	<p><i>(Largest part unknown)</i> Juan has 5 marbles. Dev has 8 more than Juan. How many marbles does Dev have?</p> <table border="1" data-bbox="1509 321 1866 436"> <tr> <td colspan="2">Dev ?</td> </tr> <tr> <td>Juan 5</td> <td>8</td> </tr> </table>	Dev ?		Juan 5	8
Dev 13															
Juan 5	?														
Dev 13															
Juan ?	5														
Dev ?															
Juan 5	8														

Introduction of bar models for multiplicative reasoning: Year 1/ 2

<p>Part-whole problems Present calculations in this way sometimes.</p>	<p><i>(Whole unknown)</i> Pencils cost 12p each. How much do 4 pencils cost?</p> <table border="1" data-bbox="543 734 900 849"> <tr> <td colspan="4">?</td> </tr> <tr> <td>12p</td> <td></td> <td></td> <td></td> </tr> </table>	?				12p				<p><i>(Value of one part unknown)</i> Dev bought 4 pencils for 48p. How much does 1 pencil cost?</p> <table border="1" data-bbox="1024 734 1381 849"> <tr> <td colspan="4">48p</td> </tr> <tr> <td>?</td> <td></td> <td></td> <td></td> </tr> </table>	48p				?				<p><i>(Number of parts unknown)</i> Pencils cost 12p each. Dev bought some pencils for 48p. How many pencils did he buy?</p> <table border="1" data-bbox="1491 734 1848 849"> <tr> <td colspan="2">48p</td> </tr> <tr> <td>12p</td> <td></td> </tr> </table>	48p		12p	
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Introduction of bar models for additive reasoning: Year 3/4

Check on mastery of additive reasoning problems from KS1, revising as necessary.

Two step joining / separating problems

Pose problems using a variety of combinations, for example:

Dev has 5 marbles. Juan has 3 more than Dev. How many marbles do they have altogether?

Juan ?	
Dev 5	3



?	
Dev 5	Juan 8

Dev has 5 marbles and Juan has 8 marbles. Ken says, "I have double the number of marbles that you have together." How many marbles does Ken have?

?	
Dev 5	Juan 8



Ken	
D+J 13	

Introduction of bar models for multiplicative reasoning: Year 3/4

Check on mastery of multiplicative reasoning problems from KS1, revising as necessary.

<p>Comparing problems</p>	<p><i>(Larger quantity unknown)</i> Bob picked 6 apples. Sue picked four times as many apples as Bob. How many apples did Sue pick?</p> <div data-bbox="543 464 900 581"> <table border="1" style="width: 100%; text-align: center;"> <tr><td colspan="4">Sue ?</td></tr> <tr><td>Bob 6</td><td></td><td></td><td></td></tr> </table> </div>	Sue ?				Bob 6				<p><i>(Smaller quantity unknown)</i> Sue picked 24 apples. She picked four times as many apples as Bob. How many apples did Bob pick?</p> <div data-bbox="1024 464 1381 581"> <table border="1" style="width: 100%; text-align: center;"> <tr><td colspan="4">Sue 24</td></tr> <tr><td>Bob ?</td><td></td><td></td><td></td></tr> </table> </div>	Sue 24				Bob ?				<p><i>(Multiplier unknown)</i> Sue picked 24 apples. Bob picked 6 apples. How many times as many apples did Sue pick than Bob?</p> <div data-bbox="1503 464 1860 581"> <table border="1" style="width: 100%; text-align: center;"> <tr><td colspan="2">Sue 24</td></tr> <tr><td>B 6</td><td style="text-align: right;">→</td></tr> </table> </div>	Sue 24		B 6	→	
Sue ?																								
Bob 6																								
Sue 24																								
Bob ?																								
Sue 24																								
B 6	→																							
<p>Fractions problems Present calculations in this way sometimes.</p>	<p><i>(Value of one part unknown)</i> What is $\frac{1}{3}$ of 21?</p> <div data-bbox="543 789 816 906"> <table border="1" style="width: 100%; text-align: center;"> <tr><td colspan="3">21</td></tr> <tr><td>?</td><td></td><td></td></tr> </table> </div>	21			?			<p><i>(Whole unknown, one part known)</i> $\frac{1}{3}$ of a number is 7. What is the number?</p> <div data-bbox="1024 789 1297 906"> <table border="1" style="width: 100%; text-align: center;"> <tr><td colspan="3">?</td></tr> <tr><td>7</td><td></td><td></td></tr> </table> </div>	?			7			<p><i>(Whole unknown, more than one part known)</i> $\frac{2}{3}$ of a number is 14. What is the number?</p> <div data-bbox="1503 789 1776 964"> <table border="1" style="width: 100%; text-align: center;"> <tr><td colspan="3">?</td></tr> <tr><td></td><td></td><td></td></tr> <tr><td colspan="2">14</td><td></td></tr> </table> </div>	?						14		
21																								
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	<p><i>(Value of one part unknown)</i> What is $\frac{2}{3}$ of 21?</p> <div data-bbox="543 1094 816 1269"> <table border="1" style="width: 100%; text-align: center;"> <tr><td colspan="3">21</td></tr> <tr><td></td><td></td><td></td></tr> <tr><td colspan="2">?</td><td></td></tr> </table> </div>	21						?																
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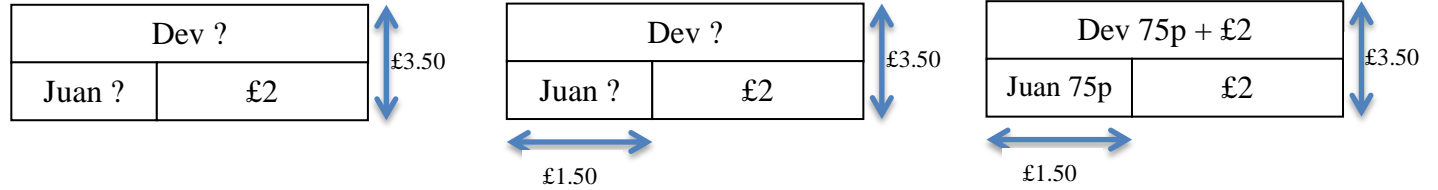
Introduction of bar models for additive reasoning: Year 5/6

Check on mastery of additive reasoning problems from KS1, revising as necessary.
Check on mastery of two step additive reasoning from Year 3/4, revising as necessary.

Comparing problems

(Difference known, total known, parts unknown)

Dev has £2 more than Juan and together they have £3.50. How much does each person have?



Introduction of bar models for multiplicative reasoning: Year 5/6

Check on retention of part-part-whole, comparing and fractions problems from Year 3/4, revising as necessary.

Multi-step part-whole, comparing and fractions problems
 Pose problems using a variety of combinations, for example:

There are 5 people living in each of the 6 houses on Green Street. $\frac{3}{5}$ of these people are children and the rest are adults. How many adults live on Green Street?


Comparing problems
 For example:


The sum of two numbers is 36. The larger number is 3 times the smaller number. What are the two numbers?

Lisa had 1750 stamps. Minah had 480 fewer stamps than Lisa. Lisa gave some stamps to Minah. Now Minah has 3 times as many stamps as Lisa. How many stamps did Minah have at first? How many stamps does Lisa have now?

There are $\frac{3}{5}$ as many boys as girls. If there are 75 girls, how many boys are there?

Ratio problems	<p><i>(Whole known, value of one part unknown)</i></p> <p>To make green paint, you need yellow and blue in the ratio of 1:2. How much yellow paint is needed to make 21 liters of green?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td colspan="3" style="text-align: center;">21</td></tr> <tr><td style="text-align: center;">Y ?</td><td style="text-align: center;">B</td><td style="text-align: center;">B</td></tr> </table>	21			Y ?	B	B	<p><i>(Whole unknown, value of one part known)</i></p> <p>To make green paint, you need yellow and blue in the ratio of 1:2. How much green paint can be made from 7 liters of yellow?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td colspan="3" style="text-align: center;">?</td></tr> <tr><td style="text-align: center;">Y 7</td><td style="text-align: center;">B</td><td style="text-align: center;">B</td></tr> </table>	?			Y 7	B	B	<p><i>(Whole unknown, value of more than one part known)</i></p> <p>To make green paint, you need yellow and blue in the ratio of 1:2. How much green paint can be made from 14 liters of blue?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td colspan="3" style="text-align: center;">?</td></tr> <tr><td style="text-align: center;">Y</td><td style="text-align: center;">B</td><td style="text-align: center;">B</td></tr> <tr><td colspan="2"></td><td style="text-align: center;">14</td></tr> </table>	?			Y	B	B			14
21																								
Y ?	B	B																						
?																								
Y 7	B	B																						
?																								
Y	B	B																						
		14																						

Multi-step additive and multiplicative reasoning problems Pose problems using a variety of combinations, for example:	<p>A jacket costs 3 times as much as a pair of shoes. If the pair of shoes costs £56, what is the total cost of the jacket and the pair of shoes?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td colspan="3" style="text-align: center;">£56</td></tr> <tr><td style="text-align: center;">S</td><td></td><td></td></tr> <tr><td></td><td></td><td></td></tr> <tr><td colspan="3" style="text-align: center;">J</td></tr> </table> <div style="text-align: center; margin-left: 100px;">  </div>	£56			S						J		
£56													
S													
J													

<p>Susan has 400 sheets of paper. She gives 5 sheets of paper to every student in a class. There are 62 students in the class. How many sheets of paper has she left?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td style="text-align: center;">62</td><td></td><td></td><td></td><td></td></tr> <tr><td colspan="5" style="text-align: center;">?</td></tr> </table> <div style="text-align: center; margin: 0 20px;">  </div> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td colspan="2" style="text-align: center;">400</td></tr> <tr><td style="text-align: center;">310</td><td style="text-align: center;">?</td></tr> </table>	62					?					400		310	?
62														
?														
400														
310	?													

<p>Year 2 collect 65 books for the book drive. They put some of the books in boxes and have 25 books left to pack. Year 3 have 4 times as many books in boxes. There are none left to pack. How many books does Year 3 collect?</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td colspan="2" style="text-align: center;">65</td></tr> <tr><td style="text-align: center;">?</td><td style="text-align: center;">25</td></tr> </table> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div style="text-align: center;"> <p>Year 2</p> <table border="1"> <tr><td style="text-align: center;">40</td></tr> </table> </div> <div style="text-align: center;"> <p>Year 3</p> <table border="1"> <tr><td></td><td></td><td></td><td></td></tr> </table> </div> </div>	65		?	25	40				
65									
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